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Fin efficiency and mechanisms of heat exchange through fins in multi-stream plate–fin heat exchangers: formulation

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Abstract—The ideal conditions underlying various methods proposed in literature for the thermal design of multi-stream plate–fin heat exchangers are discussed. The basic differential equation for heat transfer from a fin connected to two walls, when the fin bases are at unequal temperatures is solved. Two general situations when $T_A > T_B > T_0$ and $T_A > T_0 > T_B$ are investigated. The possible mechanisms, with respect to fin efficiency, of heat exchange in a plate–fin exchanger passage are examined and the relevant heat transfer equations are derived. The significance of the mechanisms is discussed. It has been shown that the equations for heat transfer across fin bases remain identical for all mechanisms. The relations between various dimensionless factors involved are presented in the form of graphs. The significance of the findings and their use in the development of a general method for rating multi-stream plate–fin heat exchangers are elucidated.

1. INTRODUCTION

The plate–fin heat exchangers used in cryogenic gas processing, aerospace and HVAC industries occupy a unique position among heat transfer equipment because of their high efficiency and multi-functionality. Considerable literature exists on various aspects of design of plate–fin heat exchangers [1–4]. The effectiveness-NTU method has been successfully applied to the design of plate–fin exchangers handling two streams since Kays and London [5]. Attempts have been made to extend this elegant method to the multi-stream case (e.g. Aulds and Barron [6]). However, because of small temperature differentials and large property variations experienced by streams in typical multi-stream plate–fin heat exchangers, the use of closed form solutions is not appropriate to the design of these exchangers [7–9]. In addition, the necessity to stack passages of various hot and cold streams so as to achieve an optimum thermal performance introduces an additional and crucial factor in the design of these exchangers. Only a few authors have addressed the problem of stacking arrangement. Fan [10], and Suessmann and Mansour [11] have addressed the sizing aspect of this problem, i.e. passage allocation, while Chato *et al.* [12], Kao [13], Weimer and Hartzog [14] and recently Prasad [15] addressed the problem of rating a multi-stream exchanger with a given stacking pattern (see [15] for a discussion of the methods employed by the above authors).

Attempts to develop suitable design methods for multi-stream exchangers have generally followed two different, but not mutually exclusive, philosophies. The first of these has been concerned with reducing the complications arising in the multi-stream case by considering ideal conditions such as a constant surface temperature [7–9, 16], or identical thermal behaviour of the passages of a given stream [12]. The second, and more realistic, line of thought treated the exchangers as true multi-passage exchangers, but made simplifying conditions about the mechanisms of heat transfer through fins [12, 14, 15]. The implications of using these idealizations have been discussed in the following sections.

2. THE CONSTANT SURFACE TEMPERATURE IDEALIZATION

According to this idealization, the temperature of the heat transfer surface is considered to be constant across all passages at any given transverse section of a multi-stream heat exchanger. Usage of this idealization effectively reduces the basic thermohydraulic calculations to a two-stream case (consisting of only a 'hot', and a 'cold', sides), but allows the enthalpy transfer calculations to be done in a truly multi-stream fashion. A few rating methods [7, 9, 16] and at least one differential sizing method [8] have used this idealization. Rating methods developed using this condition give reasonably realistic temperature and pressure profiles of the streams at a very low computational overhead as compared to more general methods [9]. However, the assumption of a constant surface temperature precludes the effects of stacking [13, 15]. Prasad [15] has shown that methods based on the constant surface temperature can lead to significant under-

NOMENCLATURE

- A_p primary surface area of passage [m^2]
- C capacity rate of passage = $[\dot{m} \cdot c_p]$ [W K^{-1}]
- c_p specific heat of stream in passage [J kg^{-1} K^{-1}]
- h heat transfer coefficient of stream [W m^{-2} K^{-1}]
- k thermal conductivity of fin [W m^{-1} K^{-1}]
- l fin length [m]
- m factor = $\sqrt{(2h/kt)}$ [m^{-1}]
- $\dot{m}$ mass flow rate of stream [kg s^{-1}]
- n number of passages in exchanger
- N number of sections in exchanger
- p a positive number
- q quantity of heat transferred to/from fin section or entire fin; also quantity of heat entering/leaving subscripted fin base (always positive when leaving fin base, and negative when entering fin base) [W m^{-1}]
- Q net quantity of heat transferred by both primary and secondary surfaces across a separating surface [W m^{-1}]
- r ratio of temperature differentials $= \theta_B/\theta_A$ at fin bases
- t fin thickness [m]
- T temperature [K]
- x distance from fin base A [m]
- X distance from fin base A of the point of extremum or point of null temperature differential, depending on case [m].

Greek symbols

- η fin efficiency defined for half-fin-length
- θ temperature differential between surface and surrounding medium [K]
- Ω defined in Section 6
- Ψ transverse heat conduction parameter, $-\sqrt{(r^2 - 2 \cdot r \cdot \cosh ml + 1)}$.

Subscripts

- A value for the A section of fin
- B value for the B section of fin
- fb value at fin base
- in entry value
- out exit value
- i passage number, also surface number
- 1/2 half-fin-length value
- s surface
- T total value for the entire fin
- x value at distance x from fin base A
- X value at the point X (at distance X from fin base A), of extremum or point of null temperature differential, depending on case.

design because they project an optimistic performance; hence use of such methods would be of limited practical utility in designing heat exchangers.

3. IDENTICAL PASSAGE BEHAVIOUR IDEALIZATION

This idealization holds that all passages through which a given stream flows show an identical thermal behaviour [12]. This is an improvement over the constant surface temperature idealization in that, it permits both thermohydraulic and enthalpy transfer calculations to be done in a true multi-stream fashion. However, the lumping of passages of each stream, implicit in the method, still precludes the effects of stacking, and further implies an ideal apportionment of hot and cold streams for heat exchange [15]; both simplifications can introduce significant error into the calculations.

4. THE HALF-FIN-LENGTH IDEALIZATION

The only way to correctly predict the performance of a multi-stream exchanger is to utilise a general multi-passage, rather than a multi-stream model [9, 16]. Authors such as Chato *et al.* [12], Weimer and Hartzog [14], and Prasad [15] have employed such models. To simplify the formulation of these models, they have used the half-fin-length idealization. According to this idealization, half the fin length in any passage is considered to be exchanging heat with the adjacent passage on either side. As will be shown in the following sections, this is strictly true only in situations where the temperature differentials at both fin bases are equal, i.e. for a passage whose separating surfaces are both at the same temperature. It is an improvement over the previous idealization(s) in that, it recognises that separating surface temperatures can vary across the cross-section of the exchanger; it, however, does not recognise that as a consequence of these differing temperatures, unequal lengths of the fin may transfer heat to respective neighbouring passages. Since it is known that the surface temperatures in multi-stream exchangers can rarely be uniform [15], this can lead to error in fin efficiency calculations, and, more importantly, in the heat transfer calculations. In addition, transverse conduction of heat (i.e. transfer of heat by conduction between non-adjacent passages) may also have to be considered when neighbouring surfaces are at unequal temperatures [15, 16].

5. FACTORS AFFECTING THE MECHANISM OF HEAT EXCHANGE

The mechanisms of heat exchange between fin and surrounding convecting medium, and the heat exchange among neighbouring passages in a multi-stream plate–fin exchanger have been addressed by relatively fewer authors [13, 16] (throughout the present paper, the term "heat exchange mechanism" has been used to denote the particular pattern of heat transfer, as described by fin base and ambient temperatures, and fin temperature profiles in each of the cases discussed in the following sections). While the importance of transverse conduction has been realised [16], no thorough analysis of the basic issues connected with heat transfer in multi-stream exchangers has appeared in the literature. Essentially, these may be listed as below:

(1) Under what circumstances does an extremum occur/not occur in the temperature profile of a fin connected to two walls?

(2) What parts of a fin connected to two walls conduct heat from/to either side, when each wall is at a different temperature?

(3) What would be the heat transfer to/from the fin?

(4) When does transverse conduction of heat take place through a fin?

It is obvious that any reliable method for either sizing or rating of a multi-stream plate–fin exchanger must take the above factors fully into consideration. The following sections endeavour to answer the above questions.

The following assumptions have been made in deriving the relations in this and the following sections:

(1) The temperature of the fin does not significantly vary through its thickness (the "thin fin" assumption).

(2) The ambient stream temperature is constant over the fin surface.

(3) The heat transfer coefficient is constant over the fin surface.

Huang and Shah [17] have presented a thorough analysis and discussion of the above and other assumptions normally used in extended surface heat transfer. From their conclusions it is evident that for the conditions obtained in most multi-stream plate–fin heat exchangers ($Bi \ll 1$, $10 < l/t < 500$, $\eta \geqslant 80\%$) the error introduced by the above assumptions would not exceed 3–5% in terms of the fin efficiency. Keeping in view the typical accuracy of the basic correlations used for the calculation of the heat transfer coefficient (5–10%), and the thermophysical properties of the process streams themselves (up to 5%), this is an acceptable error.

6. TEMPERATURE PROFILE OF A FIN IN A PLATE–FIN EXCHANGER

The problem of heat transfer from a fin in a given passage of a plate–fin heat exchanger is similar to

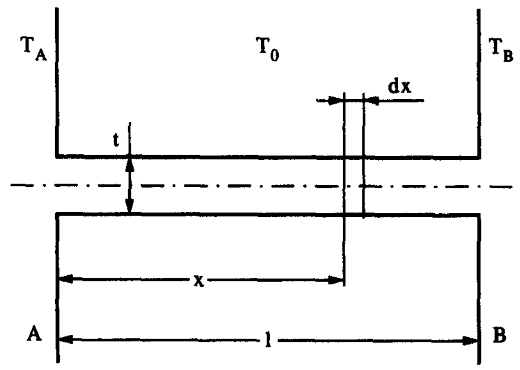

Fig. 1. Heat transfer in fin with fin bases at unequal temperatures.

the general case of convective heat transfer from a rectangular fin to an isothermal surrounding medium, for which the theory is well known [18]. Figure 1 shows two parallel walls A and B, at unequal temperatures, connected by a fin of rectangular cross-section. The governing differential equation for the temperature differential between any point on the fin and the surrounding convecting medium is known to be

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \tag{1}$$

which has the solution

$$\theta_x = P\,e^{mx} + Q\,e^{-mx} \tag{2}$$

upon the application of the boundary conditions

$$(\theta_x = \theta_A \text{ at } x = 0;\quad \theta_x = \theta_B \text{ at } x = l)$$

it can be shown that

$$P = \theta_A(1-\Omega)$$

$$Q = \theta_A\Omega$$

where

$$\Omega = \frac{e^{ml} - r}{2\sinh ml}$$

from which the general solution can be written as

$$\theta_x = \theta_A[(1-\Omega)\,e^{mx} + \Omega\,e^{-mx}]. \tag{3}$$

An important property of heat transfer in a multi-stream plate–fin heat exchanger can be derived by calculating the net heat transferred from fin to the surrounding medium (for unit fin length in the longitudinal direction)

$$q_T = \int_0^l 2h\theta_x\,dx = \frac{2h}{m}(\theta_A + \theta_B)\tanh ml/2 \tag{4}$$

multiplying and dividing by $(ml/2)$,

$$q_T = hl(\theta_A + \theta_B)\eta_{l/2}. \tag{5}$$

Under the half-fin-length idealization, the total quantity of heat transferred would be

$$q_T = q_A + q_B = hl\theta_A \eta_{1/2} + hl\theta_B \eta_{1/2}. \qquad (6)$$

Thus, it is clear that the general formula for the total convective heat transfer is identical to the one derived with the half-fin-length idealization, a fact that is not immediately obvious.

Essentially two situations are possible in a plate–fin exchanger passage. In the first, the passage either gains or loses heat to both adjacent passages; this corresponds the temperature of either fin base being higher (i.e. $T_A > T_0$ and $T_B > T_0$) or lower (i.e. $T_A < T_0$ and $T_B < T_0$) than the medium in the passage. In the second the passage may be gaining heat from one adjacent passage and losing to the other, or vice versa. In this case, the medium will have a temperature intermediate to the fin bases (i.e. $T_A > T_0 > T_B$ or $T_B > T_0 > T_A$). The following sections discuss these situations and certain special cases pertaining to them. In the cases discussed, T_A has been considered to be greater than T_B (i.e. $r < 1$). The converse case of $T_B > T_A$ (i.e. $r > 1$) can be reduced to the former by a reversal of the A and B surfaces.

7. CONDITION FOR A TEMPERATURE EXTREMUM IN THE FIN

Figure 2(a) shows the situation in a passage of a plate–fin exchanger where $T_A > T_B > T_0$ (i.e. r is positive); an extremum (which will be a minimum in this case) for θ_x exists at X. The condition for this can be derived by noting that at the extremum the derivative of the temperature differential with respect to length vanishes;

$$\frac{d\theta_x}{dx} = \theta_A[(1-\Omega)m\,e^{mx} - \Omega m\,e^{-mx}] = 0 \qquad (7)$$

from which

$$e^{2mX} = \frac{\Omega}{1-\Omega} = \frac{e^{ml} - r}{r - e^{-ml}}. \qquad (8)$$

The condition usually applied to two-stream exchangers [5] can be derived by setting $T_A = T_B$, i.e. $r = 1$, in equation (8), for which $X = 1/2$.

The significance of the extremum lies in the fact that it imposes an adiabatic boundary at X, across which no conduction of heat in the fin is possible. Thus in effect, the fin is divided into A (attached to surface A) and B (attached to surface B) sections, and consequently, the heat transfer from either the A or B section of the fin can be treated independently. It is clear that because of the extremum in the fin, no transverse conduction is possible.

8. HEAT TRANSFER IN FIN WITH TEMPERATURE EXTREMUM

The usual fin efficiency formula can be used to calculate the quantity of heat transferred by the A section of the fin in Fig. 2(a), since the boundary conditions match, i.e. $\theta_x = \theta_A$ at $x = 0$ and $d\theta_x/dx = 0$ at $x = X$.

$$q_A = 2Xh\theta_A n_A = 2Xh\theta_A(\tanh mX/mX).$$

Upon expanding the hyperbolic terms, substituting the value of e^{2mX} from equation (5) and some algebraic manipulation we obtain

$$q_A = \sqrt{(2hkt)}\theta_A \cdot \frac{\cosh ml - r}{\sinh ml}. \qquad (9)$$

The quantity of heat transferred as per the half-fin-length idealization is

$$q_{A,1/2} = \sqrt{(2hkt)}\theta_A \cdot \tanh(ml/2)$$
$$= \sqrt{(2hkt)}\theta_A \frac{\cosh ml - 1}{\sinh ml}. \qquad (10)$$

Proposing a ratio

$$\frac{q_A}{q_{A,1/2}} = \frac{\cosh ml - r}{\cosh ml - 1} \qquad (11)$$

it may be noted that this ratio becomes unity only when $r = 1$. This confirms that the half-fin-length idealization is true only when the temperature differentials at the respective fin bases are equal.

The quantity of heat transferred by the B section of the fin

$$q_B = 2(L-X)h\theta_B\eta_B = 2(1-X)h\theta_B \cdot \frac{\tanh[m(1-X)]}{m(1-X)}$$
$$= \sqrt{(2hkt)}\theta_A \cdot \frac{r \cdot \cosh ml - 1}{\sinh ml}. \qquad (12)$$

The total heat transferred can be shown to be

$$q_T = q_A + q_B = \sqrt{(2hkt)}(\theta_A + \theta_B) \cdot \tanh ml/2$$
$$= hl(\theta_A + \theta_B)\eta_{1/2}. \qquad (13)$$

Since the above quantity is identical to that predicted by equation (6), one may conclude that the half-fin-length idealization predicts the total heat transfer from/to a fin correctly, but is in error in predicting the individual A and B components, where the error depends on how much the effective fin length X (or $1-X$) differs from 1/2.

The quantities of heat leaving the respective fin bases can be calculated (in this and subsequent calculations, heat leaving the fin base is considered positive, and that entering the fin base is considered negative)

$$q_{A,fb} = -kt\left|\frac{d\theta_x}{dx}\right|_{x=0} = \sqrt{(2hkt)}\theta_A \cdot \frac{\cosh ml - r}{\sinh ml} \qquad (14)$$

$$q_{B,fb} = kt\left|\frac{d\theta_x}{dx}\right|_{x=1} = \sqrt{(2hkt)}\theta_A \cdot \frac{r \cdot \cosh ml - 1}{\sinh ml} \qquad (15)$$

from which it can be seen that $q_{A,fb} = q_A$ and $q_{B,fb} = q_B$.

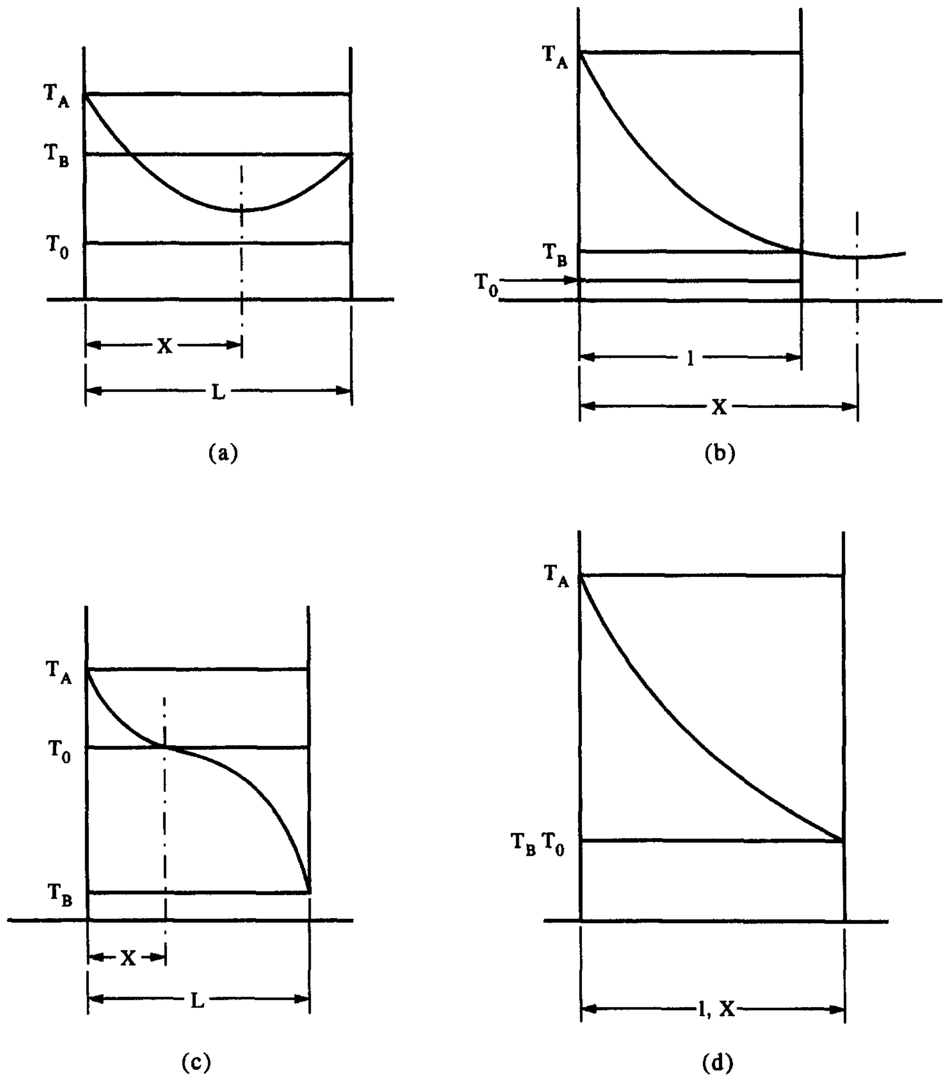

Fig. 2. Heat transfer from fin with (a) $T_A > T_B > T_0$, temperature extremum in fin; (b) $T_A > T_B > T_0$, temperature extremum outside fin; (c) $T_A > T_0 > T_B$ and (d) special case of $r = 0$.

9. WHEN THE EXTREMUM LIES OUTSIDE THE FIN

It should be noted that it is possible for the extremum to lie outside the physical boundaries of the fin. This situation is depicted by Fig. 2(b), for r less than unity. From equation (8), one can infer that when r is less than unity, $X = 1$ (i.e. the extremum occurs at fin base B) when $r = 1/\cosh ml$, and further increases (i.e. the extremum goes beyond fin base B) to reach an infinite value as r is further reduced to e^{-ml}, beyond which X becomes imaginary. Similarly, when r is greater than unity, $X = 0$ at $r = \cosh ml$ and $X = -\infty$ at $r = e^{ml}$.

It can be verified that equations (9) and (12) continue to apply to case shown in Fig. 2(b). However, as $r < 1/\cosh ml$, q_B will be negative, indicating a heat flow into, rather than away from, the B surface. Since this heat has had its origin in the A surface, it represents the transverse conduction between A and B surfaces. Thus, one may reach the important conclusion that while no transverse conduction is possible when the extremum in the temperature profile of the fin lies between the fin bases A and B, it definitely exists when the extremum falls outside this range. It should also be noted that equations (9) and (12) automatically take care of the transverse conduction when it is present.

10. CONDITION FOR THE ABSENCE OF A TEMPERATURE EXTREMUM IN THE FIN

Figure 2(c) depicts the case of $T_A > T_0 > T_B$ (i.e. r is negative). This situation can typically occur in a multi-stream plate–fin heat exchanger when the stream in the passage under study has a temperature intermediate to those in the adjacent passages. It is clear that in this case the value of the temperature differential θ_x itself vanishes at the intermediate point X, where $0 < X < 1$. Setting equation (3) equal to zero, we obtain

$$e^{2mX} = \frac{\Omega}{\Omega - 1} = \frac{e^{ml} - r}{e^{-ml} - r}. \qquad (16)$$

It may be noted that the fin loses heat to the surrounding medium up to the point X, and gains heat from the medium beyond X. It is evident that again the A and B sections can be separately treated for calculating the heat transfer.

The quantity of heat transferred by the A section of the fin

$$q_A = \int_0^X 2h\theta_x \, dx = \sqrt{(2hkt)}\theta_A \cdot \frac{\cosh ml - r - \Psi}{\sinh ml}. \qquad (17)$$

Similarly, the quantity of heat transferred by the B section of the fin

$$q_B = \int_X^l 2h\theta_x \, dx = \sqrt{(2hkt)}\theta_A \cdot \frac{r \cdot \cosh ml - 1 + \Psi}{\sinh ml}. \qquad (18)$$

Equations (17) and (18) indicate that expressions for individual heat transfer to medium do not remain identical between cases shown in Figs. 2(a) and 2(c), whereas the expression for total heat transfer remains the same. Ψ signifies the transverse conduction from the fin base A to fin base B. Also, q_B is negative (as r and Ψ are negative, all other terms on the right-hand side of equation (18) being positive), indicating a heat gain by the B surface.

The transverse heat conduction is also established directly by calculating the quantity of heat crossing the point of zero temperature differential, X

$$q_X = -kt\left|\frac{d\theta_x}{dx}\right|_{x=X} = -\sqrt{(2hkt)}\theta_A \cdot \frac{\sqrt{(r^2 - 2 \cdot r \cdot \cosh ml + 1)}}{\sinh ml}. \qquad (19)$$

The quantities of heat leaving/entering the respective fin bases are

$$q_{A,fb} = -kt\left|\frac{d\theta_x}{dx}\right|_{x=0} = \sqrt{(2hkt)}\theta_A \cdot \frac{\cosh ml - r}{\sinh ml} \qquad (20)$$

$$q_{B,fb} = kt\left|\frac{d\theta_x}{dx}\right|_{x=1} = \sqrt{(2hkt)}\theta_A \cdot \frac{r \cdot \cosh ml - 1}{\sinh ml}. \qquad (21)$$

It is evident that the expressions for the heat conducted across the respective fin bases have remained identical between cases shown in Figs. 2(a) and 2(c). The transverse conduction term appears only in the convective heat transfer equations (17) and (18). This fact can be used in the development of a general design theory for multi-stream plate–fin exchangers, as will be elaborated in Section 12.

Figure 2(d) depicts a limiting situation of the above case for which $T_B = T_0 < T_A$ (i.e. $r = 0$). For this situation $X = 1$ from equation (16), and the entire fin conducts heat to only one side. This situation is important, as it resembles what occurs in a multi-stream plate–fin heat exchanger of double-banks, and at the top and bottom of the exchanger. At a double-bank, particular hot or cold stream passages are repeated, with zero or negligible heat transfer between them; similarly at the top and bottom passages, the entire length of the fin must transfer heat to only one side. It is clear, however, that the above equations continue to apply to this special case also.

11. SIGNIFICANCE OF THE EQUATIONS DERIVED IN SECTIONS 7–10

Figure 3 contains plots of various dimensionless quantities involved in the heat exchange mechanisms studied. Figure 3(a) shows a plot of the dimensionless parameters (X/l) vs ml, for values of r between -1 and $+1$. (X/l) is the dimensionless location of the point of zero slope or curvature in the fin temperature profile. In the case of positive r, it is evident that as the fin efficiency increases, i.e. as $ml \to 0$, an extremum can exist within the physical boundaries of the fin (i.e. lie within the X/l range of 0 to 1) only for an increasingly narrow range of r. Thus as fin efficiency increases, transverse conduction should become an increasingly important factor in the heat transfer. Also note the sensitivity of the extremum to ml. This is in contrast to the case of negative r where the location of zero temperature differential X is almost insensitive to ml and also shows an almost inverse linear relationship to r. Also note that the curves for $r = p$ and $r = -p$ meet asymptotically as $ml \to \infty$, as can be expected from equations (8) and (16).

Figure 3(b) shows the dimensionless temperature profile of the fin for a typical value of $r = 0.5$, and illustrates the typical heat exchange behaviour shown in Figs. 2(a) and (b). It may be noted that the profile is a straight line for $ml = 0$, indicating that the fin temperature profile would be nearly linear at high fin efficiency. Further, for values of $ml < \cosh^{-1}(1/r)$ (i.e. $ml < 1.317$) the minimum falls outside the physical boundaries of the fin (i.e. x/l values of 0 and 1), indicating transverse conduction. Beyond this value of ml, clearly defined minima appear between x/l values of 0 and 1, indicating the absence of transverse conduction.

Figure 3(c) shows the dimensionless temperature profile of the fin for a typical negative $r = -0.5$, illustrating the heat exchange behaviour shown in Figs. 2(c) and (d). The point of zero temperature differential

X can be seen as a point of contraflexure in each curve, indicating that this is a point of sign change in curvature, as can be expected from equation (1). It can also be seen that the position of X has a relatively poor dependence on ml, as was seen earlier in Fig. 3(a) also.

Figure 3(d) shows the relation of the heat transfer ratio $q_{B,fb}/q_{A,fb}$ with ml for values of r ranging from -1 to $+1$. Note that for positive values of r, the value of the ratio is close to -1 at low values of ml, indicating that most of the heat from the A surface would be transferred to the B surface in transverse conduction. The ratio also exhibits an almost linear relation with r at moderate to higher values of ml, irrespective of the sign of r. This indicates that in most practical situations the quantities of heat transferred by the A and B sections of the fin are nearly proportional to the respective base temperature differ-

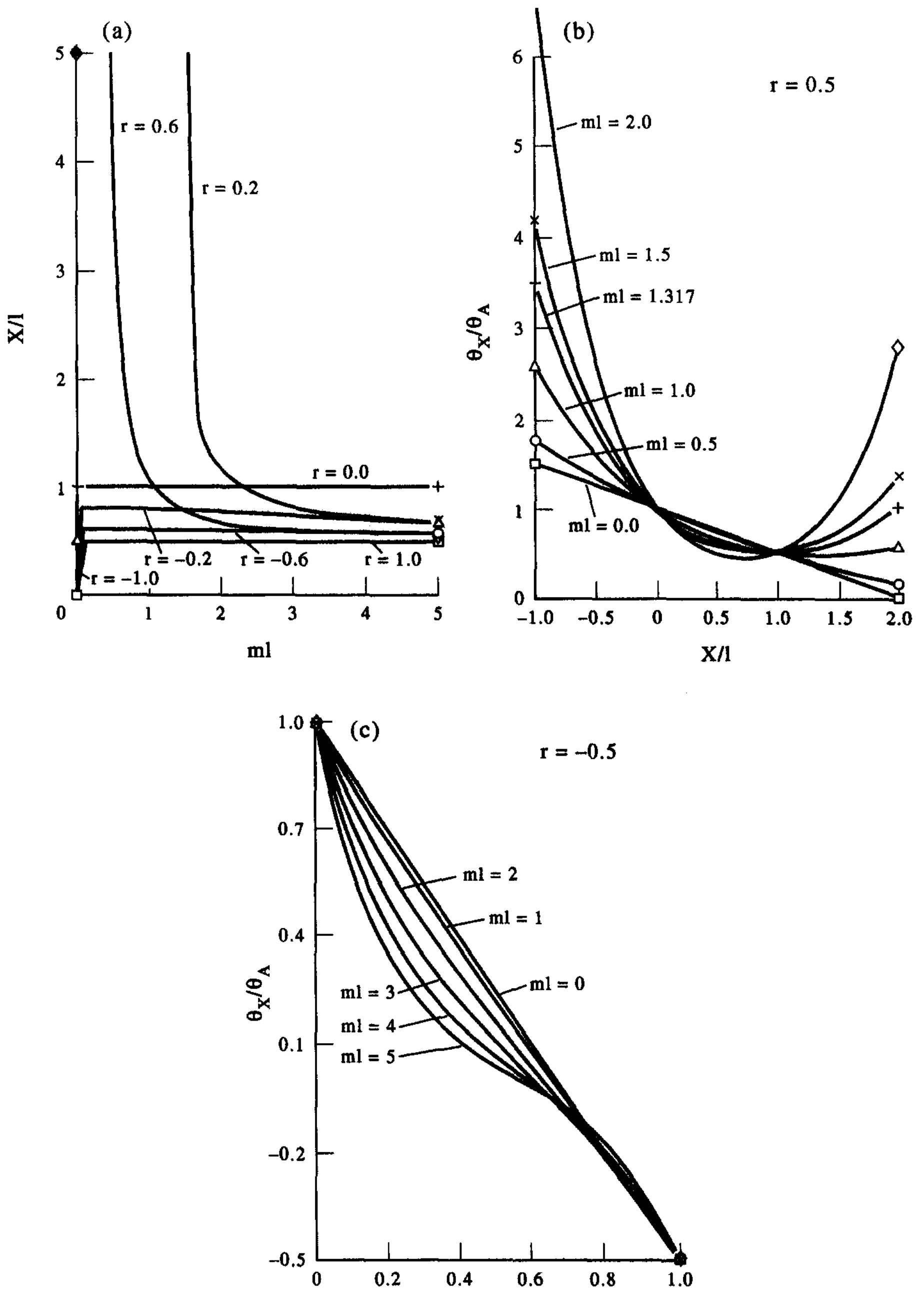

Fig. 3. (a) Dimensionless plot of X/l vs ml, at $r = -1$ to $+1$. (b) Dimensionless plot of θ_x/θ_A vs (x/l), at $r = 0.5$. (c) Dimensionless plot of θ_x/θ_A vs (x/l), at $r = -0.5$. (d) Dimensionless plot of q_B/q_A vs ml, at $r = -1$ to $+1$. (e) Dimensionless plot of $q_A/q_{A,1/2}$ vs ml, at $r = -1$ to $+1$. (f) Dimensionless plot of $q_B/q_{B,1/2}$ vs ml, at $r = -1$ to $+1$. (g) Dimensionless plot of $q_X/q_{A,fb}$ vs r, at $ml = 0$–5.

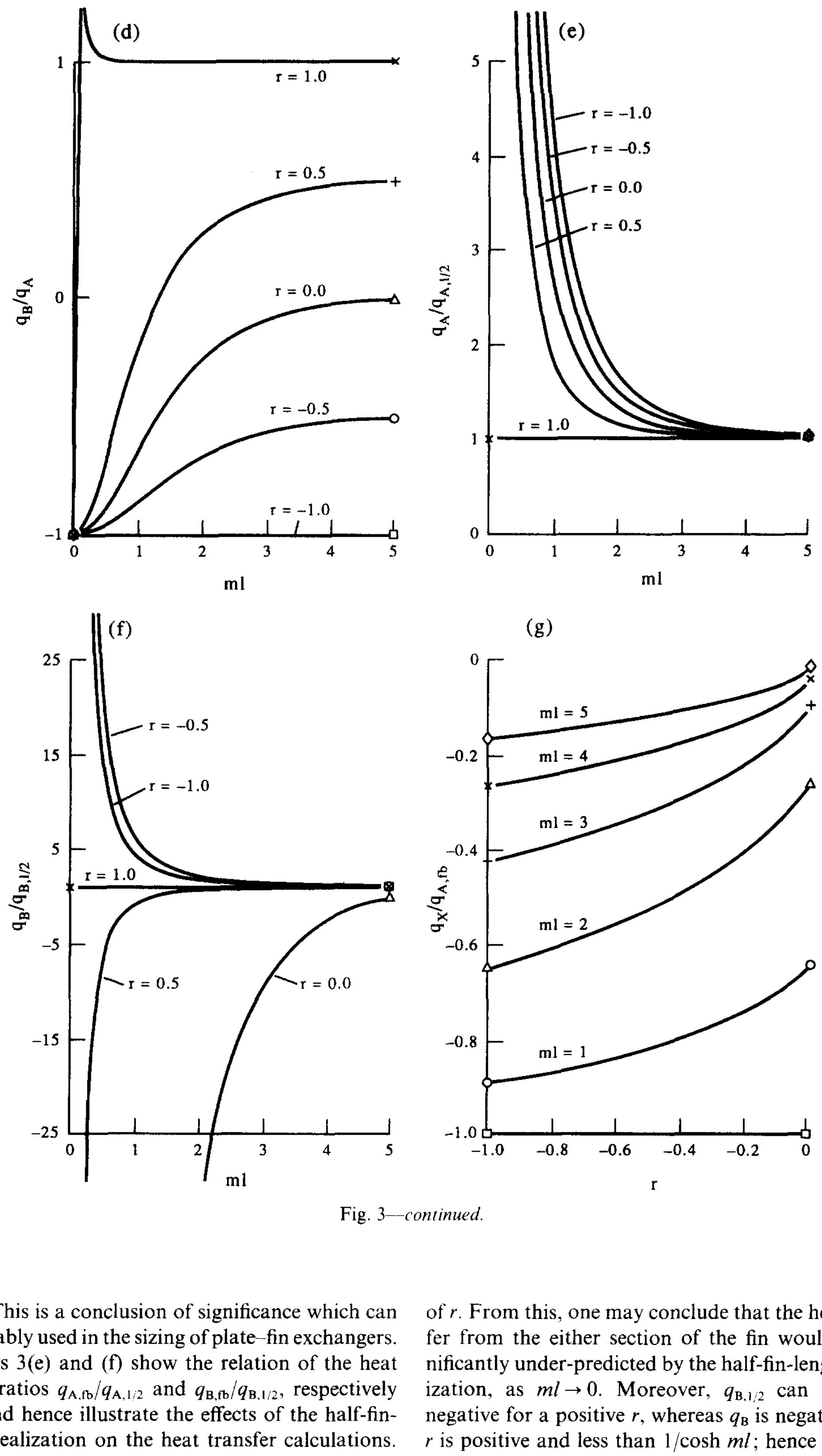

Fig. 3—*continued.*

entials. This is a conclusion of significance which can be profitably used in the sizing of plate–fin exchangers.

Figures 3(e) and (f) show the relation of the heat transfer ratios $q_{A,fb}/q_{A,1/2}$ and $q_{B,fb}/q_{B,1/2}$, respectively to ml, and hence illustrate the effects of the half-fin-length idealization on the heat transfer calculations. Note that the former ratio is significantly above unity for low to moderate values of ml (at all values of r other than $+1$). Similarly, in the latter case also, the ratio is considerably different from unity for all values of r. From this, one may conclude that the heat transfer from the either section of the fin would be significantly under-predicted by the half-fin-length idealization, as $ml \to 0$. Moreover, $q_{B,1/2}$ can never be negative for a positive r, whereas q_B is negative when r is positive and less than $1/\cosh ml$; hence the error in q_B is much more serious than that in q_A when the half-fin-length idealization is used, as there would be an error in both the sign and magnitude of the quantity of heat transferred when transverse conduction

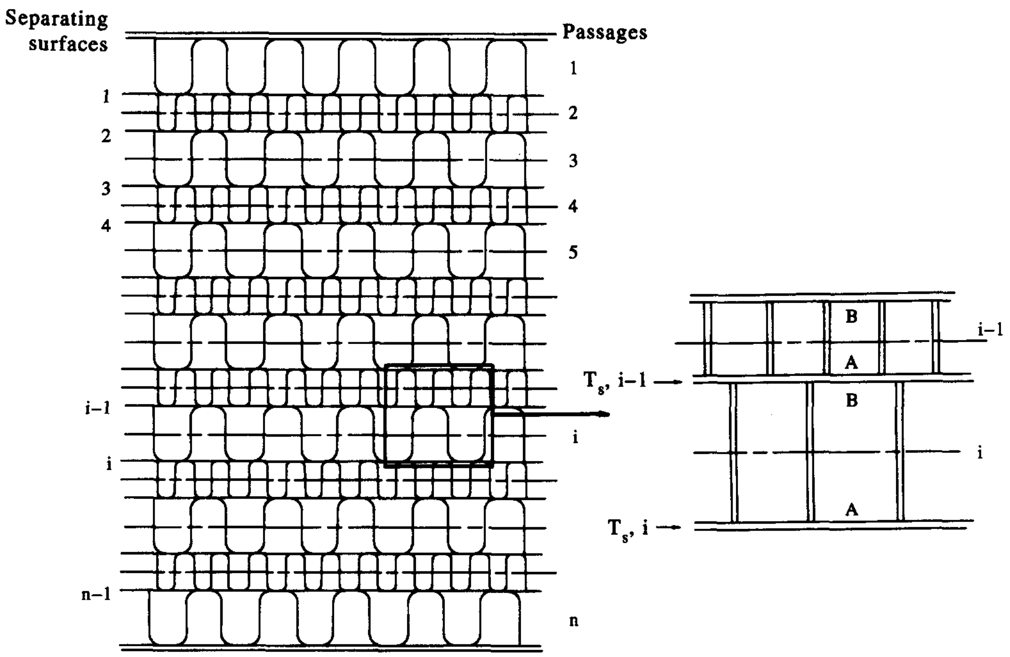

Fig. 4. Heat transfer in a section of a multi-stream plate–fin heat exchanger.

takes place. These conclusions indicate that significant errors are possible in multi-stream design schemes employing the half-fin-length idealization.

Figure 3(g) is a plot of the ratio $q_X/q_{A,fb}$ against r, for for various values ml (see Fig. 2(c)). This plot shows the effect of r and ml on extent of transverse conduction in Fig. 2(c). Note that at high fin efficiency (i.e. $ml \rightarrow 0$), most of the heat from fin base A is conducted to fin base B. This behaviour is similar to that observed earlier for positive r when transverse conduction was present [i.e. in Fig. 2(b)].

12. DEVELOPMENT OF RATING METHOD WITH DERIVED EQUATIONS

The derivations of Sections 7–10 answer the basic questions raised in Section 5. Figure 2 covers exhaustively all the heat exchange mechanisms possible in a multi-stream plate–fin exchanger. Of these, Figs. 2(b) and (d) are only special cases respectively of Figs. 2(a) and (c). It is evident that equations (14) and (15) apply rigorously to all cases represented in Fig. 2, and thus form the basis for a general design method of multi-stream plate–fin heat exchangers. As explained in detail in [15], for a multi-stream exchanger with an arbitrary arrangement of passages, the following equations can be written for a given section of the ith passage, bounded by the $(i-1)$th and the ith separating surfaces (Fig. 4):

$$Q_{A,i-1} + Q_{B,i} = 0$$

(for heat transfer across the $(i-1)$th surface) in which the terms can be further split into primary and secondary surface heat transfer

$$0.5h_{i-1}A_{p,i-1}(T_{i-1} - T_{s,i-1}) + q_{A,fb,i-1} + 0.5h_iA_{p,i}(T_i - T_{s,i-1}) + q_{B,fb,i} = 0 \quad (22)$$

and

$$Q_i = C_i(T_{out,i} - T_{in,i})$$

(for heat transfer in the ith passage) which can be elaborated to

$$0.5h_iA_{p,i}(T_i - T_{s,i-1}) + 0.5h_iA_{p,i}(T_i - T_{s,i}) + q_{T,i} = C_i(T_{out,i} - T_{in,i}). \quad (23)$$

A total of $(2n)$ of the above equations [n each of equations (22) and (23)] can be written. Since T_i can be taken as 0.5 $(T_{in,i} + T_{out,i})$, the total number of unknowns in the system are only $(2n-1)$, i.e. n exit temperatures and $(n-1)$ separating surface temperatures for an exchanger of n passages. The system can thus be solved for the exit and surface temperatures by employing a suitable matrix method. Thus, equations (22) and (23), together with equations (14) and (15) [which are identical to equations (20) and (21)], constitute a rigorous framework for the rating of multi-stream plate–fin exchangers. It may also be noted that the effects of transverse conduction are automatically take care of in this method. Since equations (22) and (23) are based on energy balance and are independent of any flow pattern, they can be applied equally well to suitably chosen sections of exchangers employing crossflow, counterflow or parallel flow. The development of a matrix rating method (employing the half-fin-length idealization) for the multi-stream counter/parallel flow exchanger has been discussed in detail in [16], and a step-by-step procedure for rating has been given. The implications of employing equations (14) and (15) instead of the half-fin-length idealization will form the subject matter of another paper, currently under preparation.

13. CONCLUSIONS

The basic differential equation for heat transfer from a fin connected to two walls, when the fin bases are at unequal temperatures is solved. Two general situations when $T_A > T_B > T_0$ and $T_A > T_0 > T_B$ are investigated. It has been found that the basic equations for heat transfer from the fin bases are identical across all mechanisms. Transverse conduction is absent only when an extremum is present within the physical boundaries of the fin in the fin temperature profile. It was found that transverse conduction assumes increasing importance as the fin efficiency increases. It has been shown that while the net heat exchange to medium inside the passage by the fin is correctly predicted by the half-fin-length idealization, considerable error could result in the calculation of the individual components of heat transfer associated with the two fin bases when this idealization is used. It has been demonstrated that a general design theory for the multi-stream exchanger, which also automatically accounts for the transverse conduction, results from the derived equations.

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